Classical Interaction Length for Two-Particle Scattering and Its Manifestation in Transmission Amplitude

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Abstract. We analyze spatial shifts arising from a classical one-dimensional collision between two particles interacting via a short-range repulsive potential, using a rectangular potential of finite spatial extent as an explicit example. The finite extent of the interaction region produces asymptotic displacements of the particles relative to freely moving reference trajectories. For reflection, these shifts admit a simple geometrical interpretation in terms of the interaction range. For transmission, the shifts arise from the temporary exchange of momentum as the particles move through the interaction region, leading to a finite lag in their separation relative to non-interacting motion, even though their initial velocities are restored after the collision. After deriving these effects classically using center-of-mass and relative coordinates, we show that they are reproduced by a quantum-mechanical wave-packet treatment of the corresponding two-particle scattering problem. In the quantum description, the same spatial shifts emerge from derivatives of the scattering phases associated with reflection and transmission amplitudes. This establishes a direct correspondence between finite-range classical collision effects and the phase structure of two-particle quantum scattering.